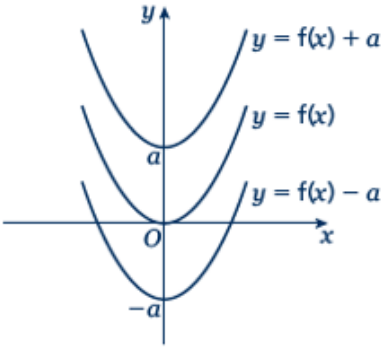
Translating graphs

A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation

Key points

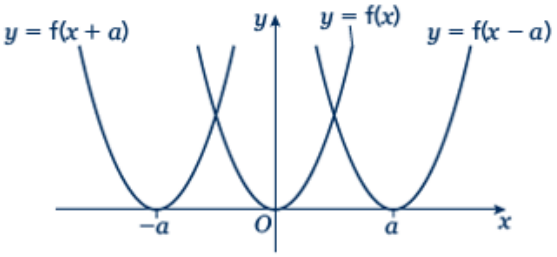
- The transformation $y = f(x) \pm a$ is a translation of $y = f(x)$ parallel to the y-axis; it is a vertical translation.

  As shown on the graph,
  - $y = f(x) + a$ translates $y = f(x)$ up
  - $y = f(x) – a$ translates $y = f(x)$ down.



- The transformation $y = f(x \pm a)$ is a translation of $y = f(x)$ parallel to the x-axis; it is a horizontal translation.

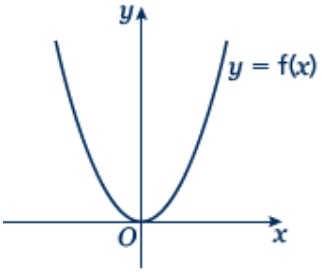
  As shown on the graph,
  - $y = f(x + a)$ translates $y = f(x)$ to the left
  - $y = f(x – a)$ translates $y = f(x)$ to the right.

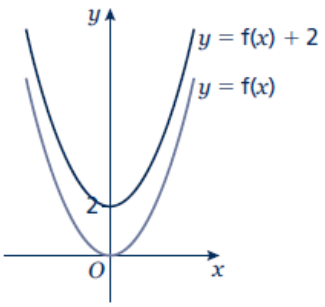


Examples

Example 1 The graph shows the function $y = f(x)$.

Sketch the graph of $y = f(x) + 2$.

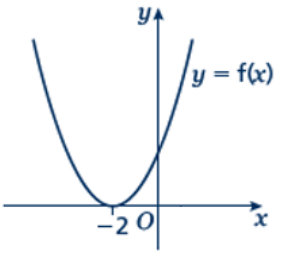


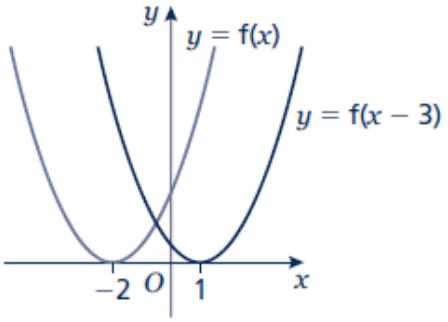


For the function $y = f(x) + 2$ translate the function $y = f(x)$ 2 units up.

Example 2 The graph shows the function $y = f(x)$.

Sketch the graph of $y = f(x - 3)$.

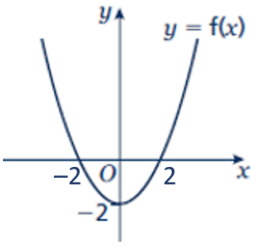




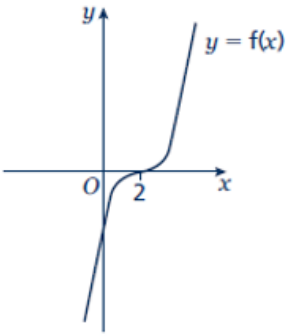
For the function $y = f(x - 3)$ translate the function $y = f(x)$ 3 units right.

Practice

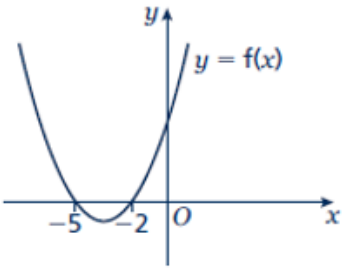
1 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.



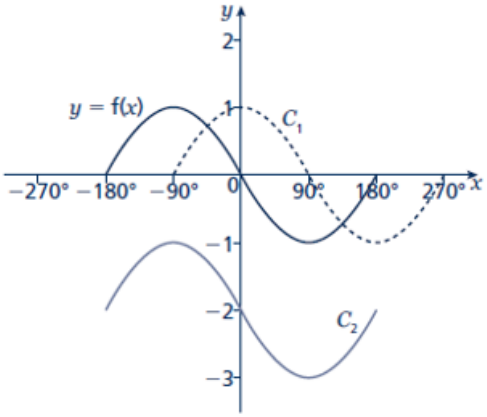
2 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x + 3)$ and $y = f(x) - 3$.



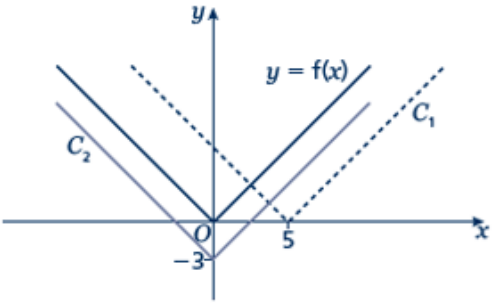
3 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch the graph of $y = f(x - 5)$.



4 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



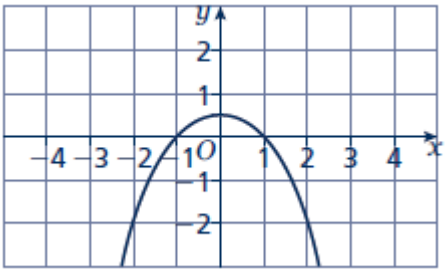
5 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



6 The graph shows the function $y = f(x)$.

a Sketch the graph of $y = f(x) + 2$

b Sketch the graph of $y = f(x + 2)$

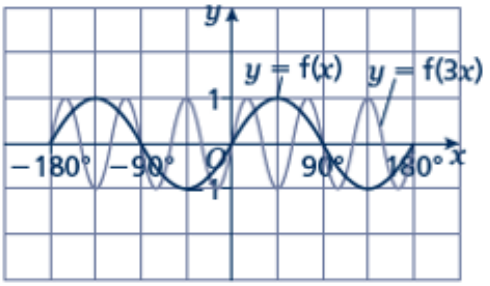


Stretching graphs

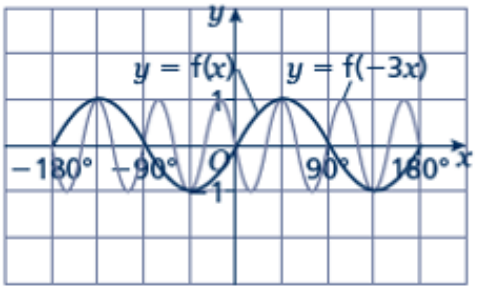
A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation
Textbook: Pure Year 1, 4.6 Stretching graphs

Key points

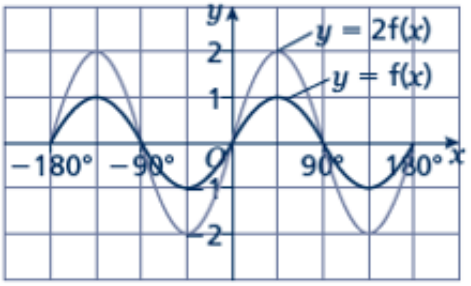
- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis.



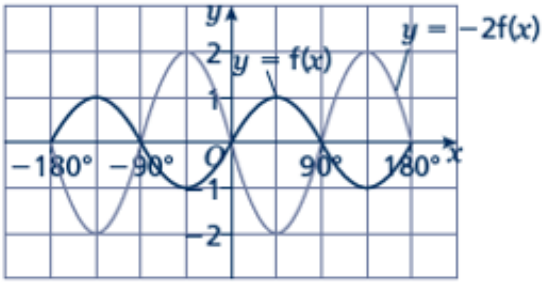
- The transformation $y = f(-ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x-axis and then a reflection in the y-axis.



- The transformation $y = af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis.



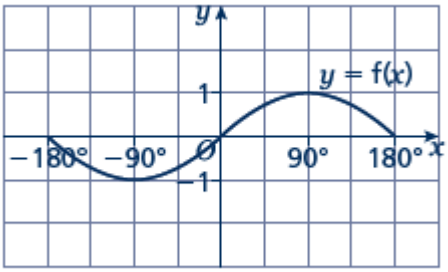
- The transformation $y = -af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y-axis and then a reflection in the x-axis.

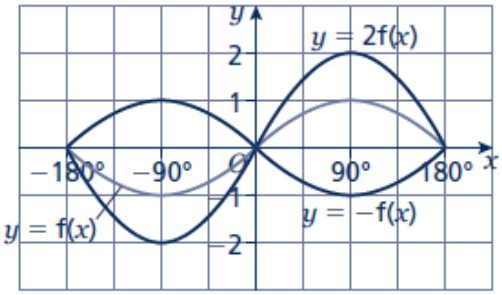


Examples

Example 3 The graph shows the function $y = f(x)$.

Sketch and label the graphs of $y = 2f(x)$ and $y = -f(x)$.



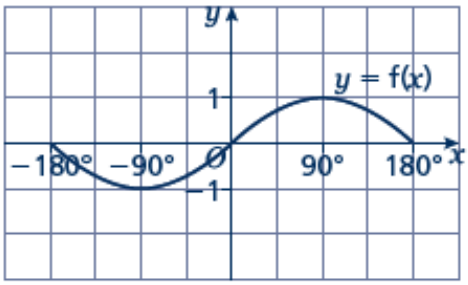


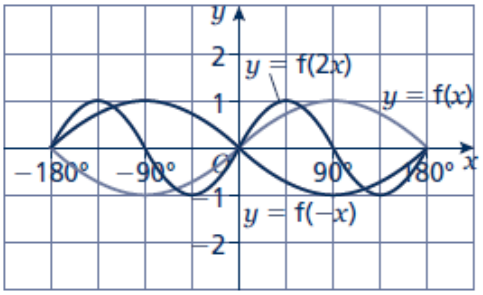
The function $y = 2f(x)$ is a vertical stretch of $y = f(x)$ with scale factor 2 parallel to the y-axis.

The function $y = -f(x)$ is a reflection of $y = f(x)$ in the x-axis.

Example 4 The graph shows the function $y = f(x)$.

Sketch and label the graphs of $y = f(2x)$ and $y = f(-x)$.





The function $y = f(2x)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{2}$ parallel to the x-axis.

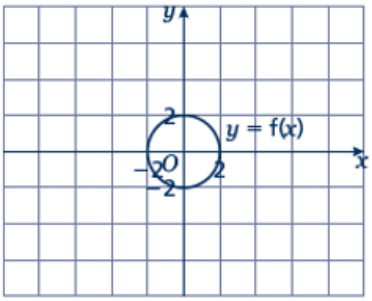
The function $y = f(-x)$ is a reflection of $y = f(x)$ in the y-axis.

Practice

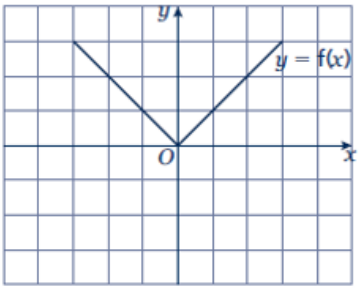
7 The graph shows the function $y = f(x)$.

a Copy the graph and on the same axes sketch and label the graph of $y = 3f(x)$.

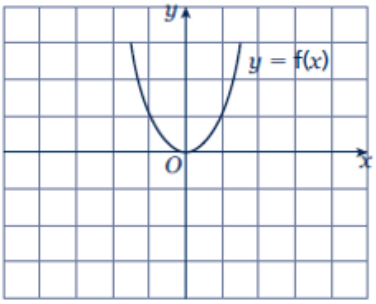
b Make another copy of the graph and on the same axes sketch and label the graph of $y = f(2x)$.



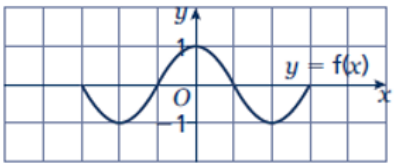
8 The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = -2f(x)$ and $y = f(3x)$.



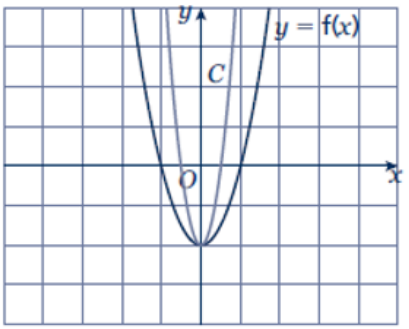
9 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch and label the graphs of $y = -f(x)$ and $y = f\left(\frac{1}{2}x\right)$.



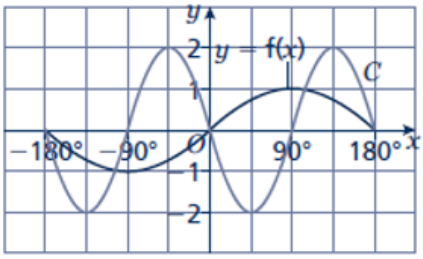
10 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch the graph of $y = -f(2x)$.



11 The graph shows the function $y = f(x)$ and a transformation, labelled C. Write down the equation of the translated curve C in function form.



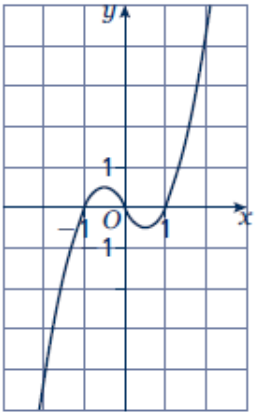
12 The graph shows the function $y = f(x)$ and a transformation labelled C.
Write down the equation of the translated curve C in function form.



13 The graph shows the function $y = f(x)$.

a Sketch the graph of $y = -f(x)$.

b Sketch the graph of $y = 2f(x)$.



Extend

14 **a** Sketch and label the graph of $y = f(x)$, where $f(x) = (x - 1)(x + 1)$.

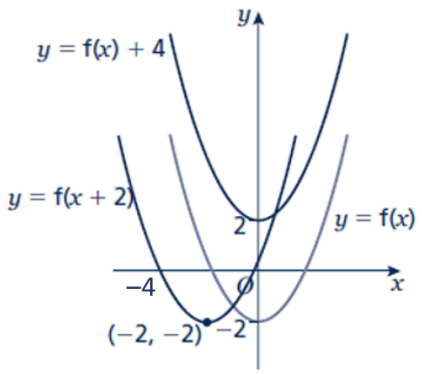
b On the same axes, sketch and label the graphs of $y = f(x) - 2$ and $y = f(x + 2)$.

15 **a** Sketch and label the graph of $y = f(x)$, where $f(x) = -(x + 1)(x - 2)$.

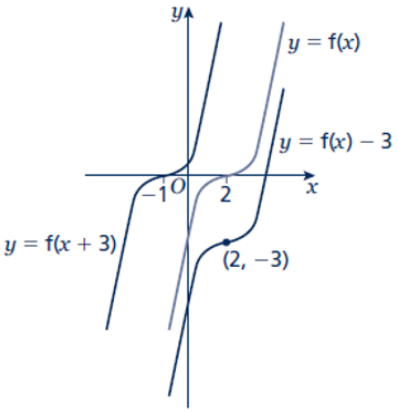
b On the same axes, sketch and label the graph of $y = f\left(-\frac{1}{2}x\right)$.

Answers

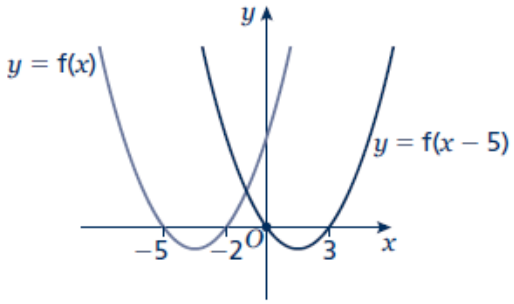
1



2



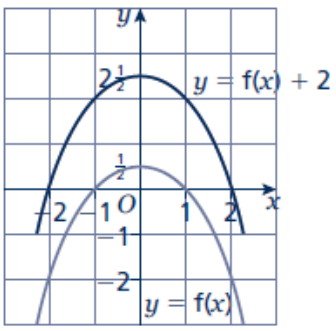
3



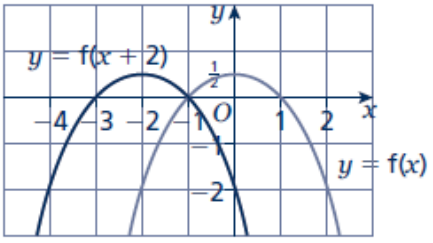
4 C_1: $y = f(x - 90°)$
C_2: $y = f(x) - 2$

5 C_1: $y = f(x - 5)$
C_2: $y = f(x) - 3$

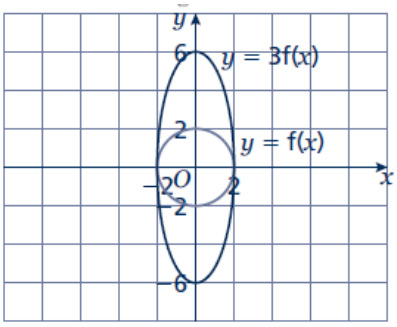
6 **a**



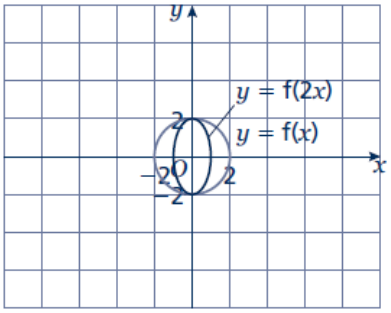
b



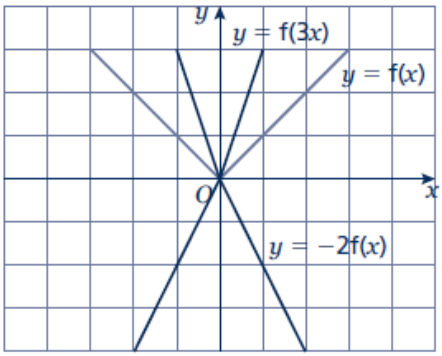
7 **a**

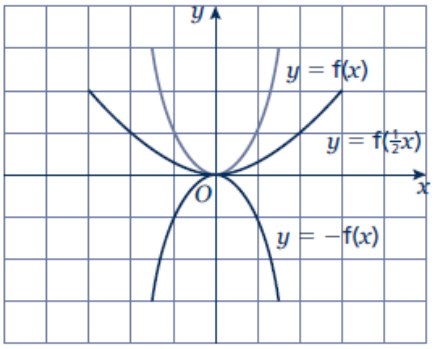


b



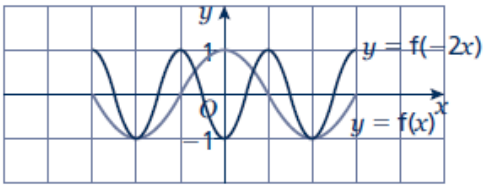
8





9

10

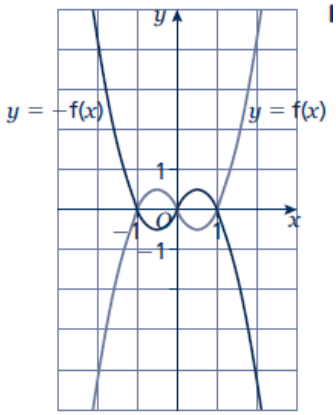


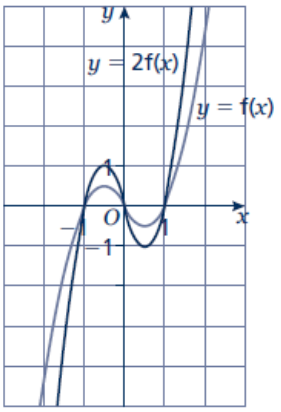
11 $y = f(2x)$

12 $y = -2f(2x)$ or $y = 2f(-2x)$

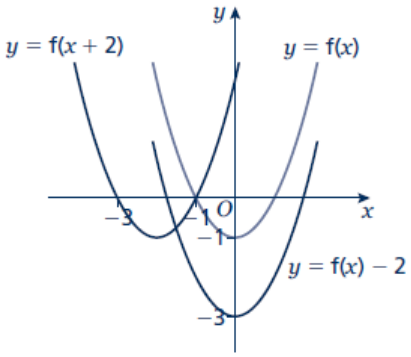
13 **a**

b

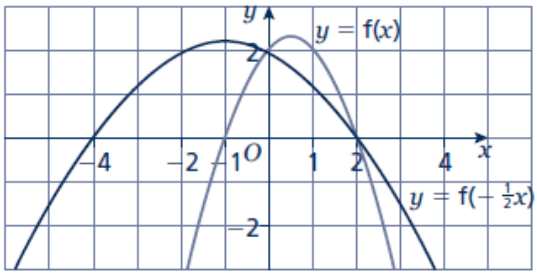

y
y = −f(x)
y = f(x)
1
O
x
−1
1
−1


y
y = 2f(x)
y = f(x)
1
O
x
−1
1
−1

14


y
y = f(x + 2)
y = f(x)
−3
−1
O
x
−1
y = f(x) − 2
−3

15


y
y = f(x)
2
−4
−2
−1
O
1
2
4
x
y = f(− ½x)
−2