

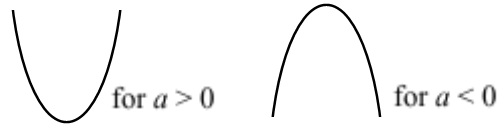
Sketching quadratic graphs

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y -axis substitute $x = 0$ into the function.
- To find where the curve intersects the x -axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.



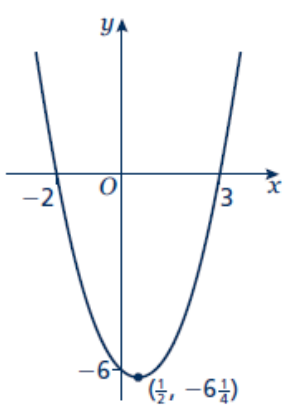

Examples

Example 1 Sketch the graph of $y = x^2$.

	<p>The graph of $y = x^2$ is a parabola.</p> <p>When $x = 0$, $y = 0$.</p> <p>$a = 1$ which is greater than zero, so the graph has the shape:</p>
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Example 2 Sketch the graph of $y = x^2 - x - 6$.

<p>When $x = 0$, $y = 0^2 - 0 - 6 = -6$ So the graph intersects the y-axis at $(0, -6)$</p> <p>When $y = 0$, $x^2 - x - 6 = 0$</p> <p>$(x + 2)(x - 3) = 0$</p> <p>$x = -2$ or $x = 3$</p>	<ol style="list-style-type: none"> 1 Find where the graph intersects the y-axis by substituting $x = 0$. 2 Find where the graph intersects the x-axis by substituting $y = 0$. 3 Solve the equation by factorising. 4 Solve $(x + 2) = 0$ and $(x - 3) = 0$.
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<p>So, the graph intersects the x-axis at $(-2, 0)$ and $(3, 0)$</p> $x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$ $= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$ <p>When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$</p> 	<p>5 $a = 1$ which is greater than zero, so the graph has the shape: </p> <p style="text-align: right;"><i>(continued on next page)</i></p> <p>6 To find the turning point, complete the square.</p> <p>7 The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.</p>
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Practice

- 1 Sketch the graph of $y = -x^2$.
- 2 Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$	b $y = x(x - 3)$	c $y = (x + 1)(x + 5)$
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- 3 Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$ **b** $y = x^2 - 5x + 4$ **c** $y = x^2 - 4$
d $y = x^2 + 4x$ **e** $y = 9 - x^2$ **f** $y = x^2 + 2x -$

3

4 Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

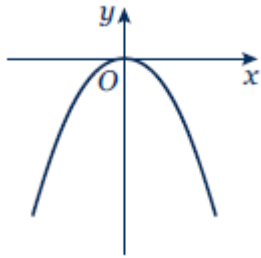
5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a $y = x^2 - 5x + 6$ **b** $y = -x^2 + 7x - 12$ **c** $y = -x^2 + 4x$

6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

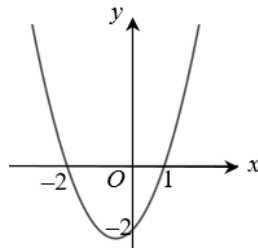
Answers

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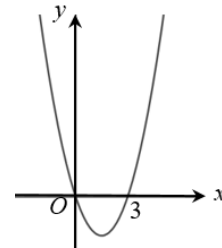


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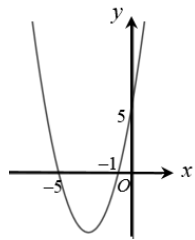
a



b

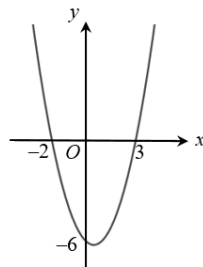


c

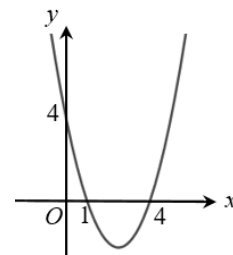


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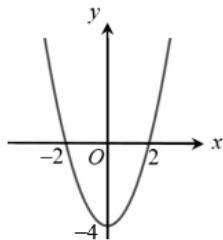
a



b



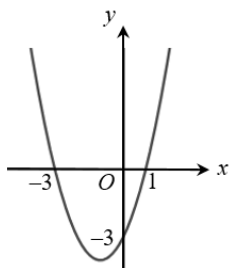
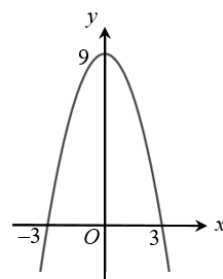
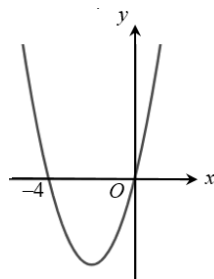
c



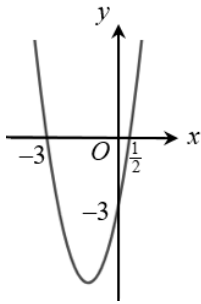
d

e

f



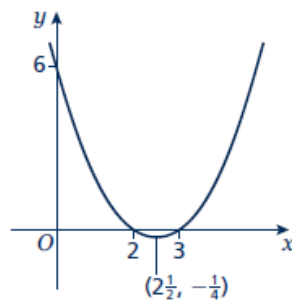
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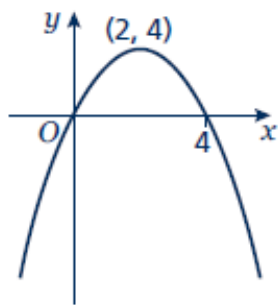
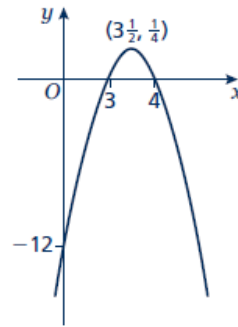
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a

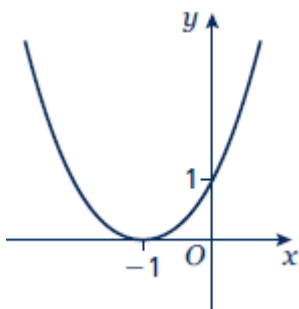
b



c



6



Line of symmetry at $x = -1$.